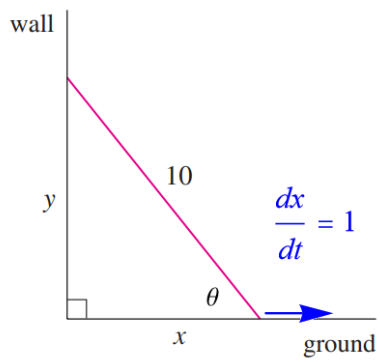
Exercise 32

How fast is the angle between the ladder and the ground changing in Example 2 when the bottom of the ladder is 6 ft from the wall?

Solution

In Example 2 there's a 10-foot ladder that's leaning against a wall and sliding on the ground at a rate of 1 ft/s.



Since we want to know $d\theta/dt$ when $x = 6$, use trigonometry to relate the angle with the known sides of the triangle.

$$\cos\theta = \frac{x}{10}$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\frac{d}{dt}(\cos\theta) = \frac{d}{dt}\left(\frac{x}{10}\right)$$

$$(-\sin\theta)\cdot\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{dx}{dt}$$

$$\left(-\frac{y}{10}\right)\cdot\frac{d\theta}{dt} = \frac{1}{10}\cdot(1)$$

Solve for $d\theta/dt$.

$$\frac{d\theta}{dt} = -\frac{1}{y}$$

$$= -\frac{1}{\sqrt{10^2 - x^2}}$$

Therefore, the rate of change of the angle when the bottom of the ladder is 6 feet from the wall is

$$\left.\frac{d\theta}{dt}\right|_{x=6} = -\frac{1}{\sqrt{10^2-(6)^2}} = -\frac{1}{8}\,\frac{\text{radians}}{\text{second}} = -0.125\,\frac{\text{radians}}{\text{second}}.$$